Economics of High Occupancy Committed Lanes

Equilibria and optimization

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Plan

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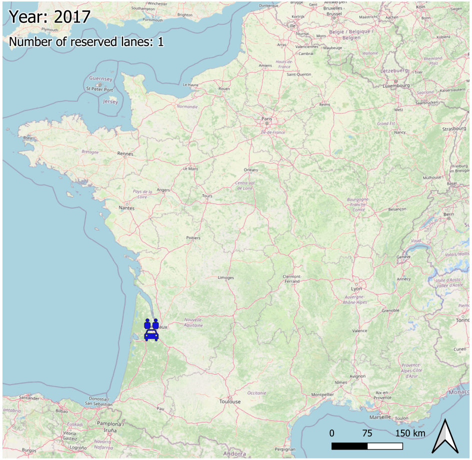
Lane reserved to carpoolers are getting common



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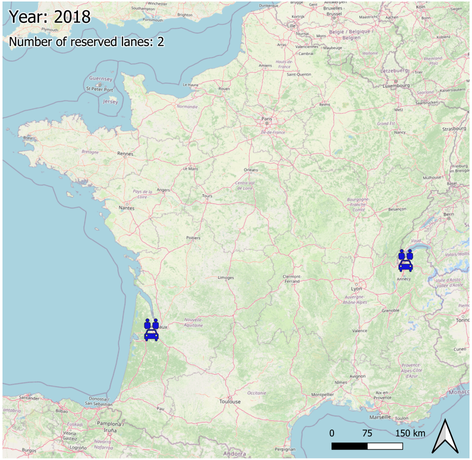

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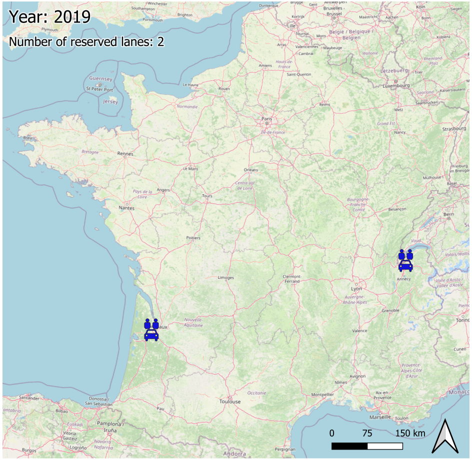

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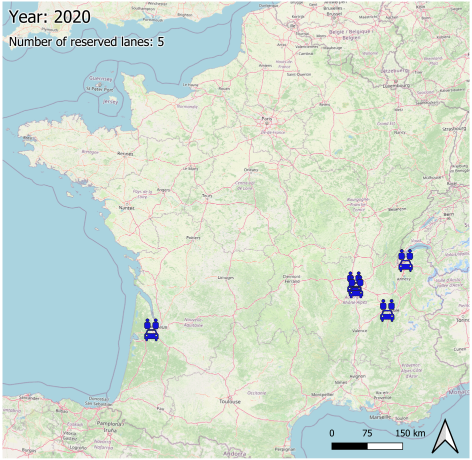

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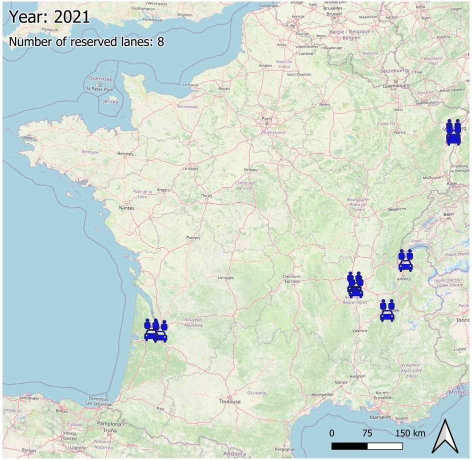

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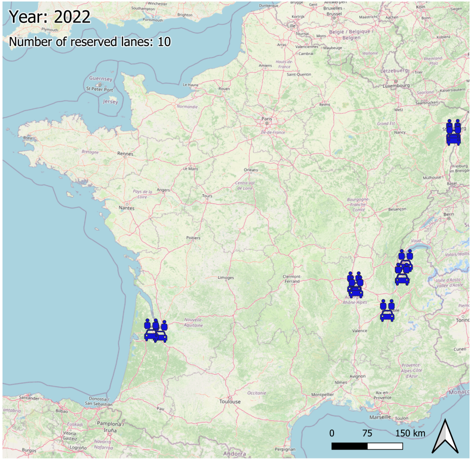

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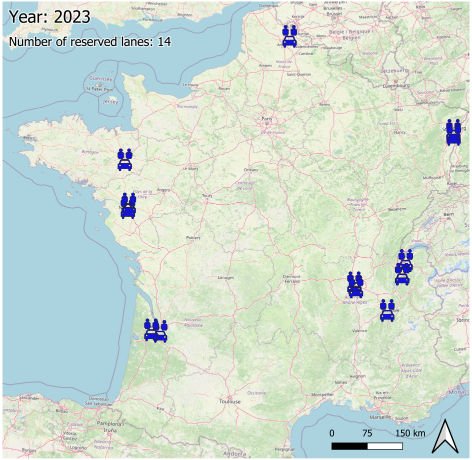

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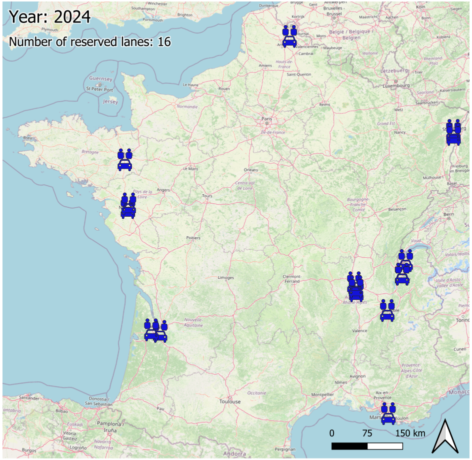

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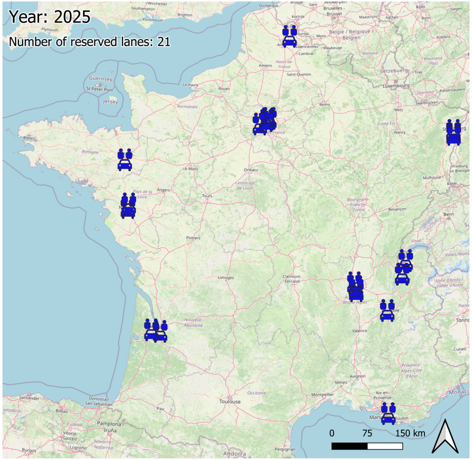

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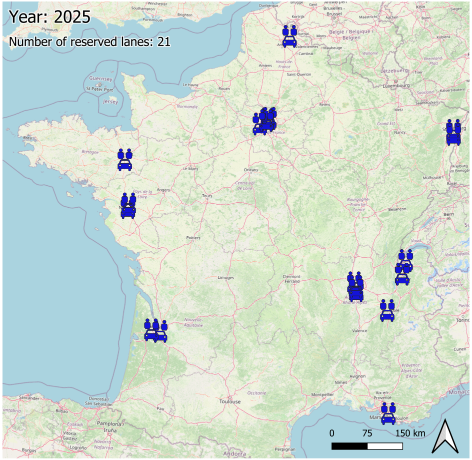

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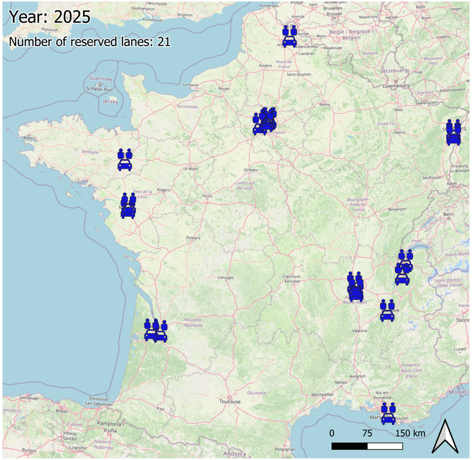

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- Knowledge about their impact on social welfare is sparse.

Research questions

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- Under which conditions is it of interest to reserve capacity to carpoolers on a highway?
- Could we imagine better reservation systems than classical HOVL?

Plan

Conveyed literature (short list)

Bottleneck, carpooling and reserved lane literature

Vickrey 1969, **Arnott et al. 1990**

Qian et al. 2011, **Liu et al. 2017**, Xiao et al. 2021

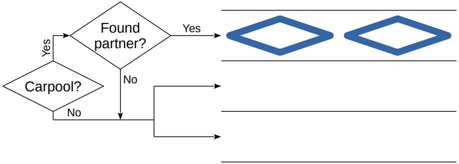
Miscellaneous carpooling literature

Yang et al. 1999, **Konishi et al. 2010**, Leurent 2024

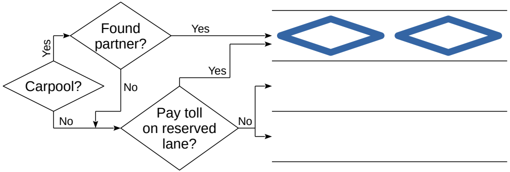
Le Goff et al. 2022

Usual capacity reservation for carpooling

HIGH OCCUPANCY VEHICLE LANE (HOVL)



HIGH OCCUPANCY VEHICLE LANE (HOTL)



Capacity reservation under study

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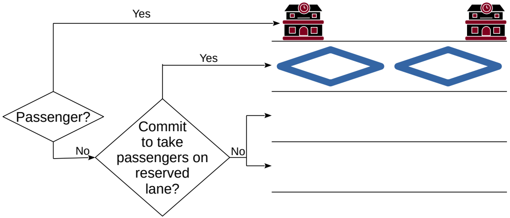

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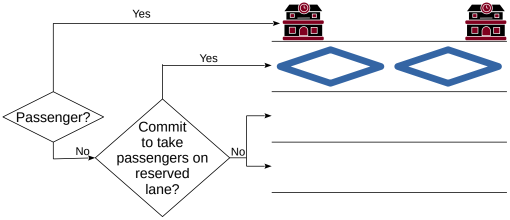

↑

This as what we explore, before comparing it to HOVL.

Mathematical framework

Parameter	Notation
Fixed population	N
3 modes (solo, cp-driver, cp-passenger)	N_s, N_d, N_p
Payment for cp-driver/cp-passenger	ρ_d, ρ_p
Amount of shared/not-shared capacity	φs, $(1-\varphi)s$
Cp-driver inconvenience cost	$\overline{\Delta}_d$
Cp-passenger inconvenience cost (distributed)	$F(\Delta_p) \in [0,1]$
Bottleneck time coefficient	δ
Car usage cost	f
Other externalities	E
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For each triplet of policy parameters (φ, ρ_d, ρ_d). If there is an equilibrium, then the resulting modal split is unique. Conditions of equilibrium existence have a closed form.

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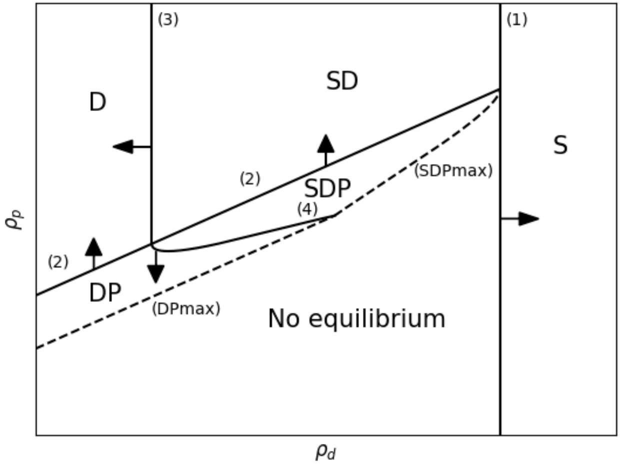
Theorem (Modal split as equilibrium)

For each "feasible" modal split ($N_s + N_d + N_p = N$, $N_p \leq MN_d$ and $N_i \geq 0 \; \forall i \in \{s, d, p\}$), there exist at least one policy $(\varphi, \rho_d, \rho_p)$ such that the "feasible" modal split is the unique resulting modal split at equilibrium.

From monitoring modal split...



ρ_d = fare paid by carpooling driver

ρ_p = fare paid by carpooling passenger

…to monitoring social cost

Hypothesis: $OCPF = 1$

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$$SC_{0<\varphi<1} = \underbrace{\frac{\delta N_s^2}{(1-\varphi)s} + \frac{\delta N_d^2}{\varphi s} + \frac{\delta N_d N_p}{\varphi s}}_{\text{Congestion cost}}$$

No ρ_p nor ρ_d...

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...still no ρ_p nor ρ_d...

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$$+ \underbrace{(N_s + N_d)f}_{\text{Car operation costs}} + \underbrace{(N_s + N_d)E}_{\text{Other car externalities}}$$

$$+ \underbrace{N_d \overline{\Delta_d}}_{\text{Cp-drivers inconvenience costs}} + \underbrace{N \int_{\Delta_p^{\min}}^{F^{-1}\left(\frac{N_p}{N}\right)} \Delta_p dF}_{\text{Cp-passengers inconvenience costs}}$$

…and no more ρ_p nor ρ_d.

Solving the problem

1. Find the “feasible” modal split that minimize social cost without accounting for the values of ρ_d and ρ_p. To do this, solve a Lagrangian for a *non-convex* problem on a convex closed bounded set.
2. Deduce the appropriate fare policy (ρ_d, ρ_p)
3. Analyze the results (either mathematically or with numerical simulations).

Interpreting fares for interior solutions

- $\rho_d = -\frac{\delta N_s}{(1-\varphi)s} + \frac{\delta(N_d+N_p)}{\varphi s}$
- $\rho_p = -\frac{\delta N_s}{(1-\varphi)s} - E$
- $\rho_p - \rho_d = -E - \frac{\delta(N_d+N_p)}{\varphi s}$

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For optima that are interior solutions, fares are compensating externalities.

Some mathematical properties

Theorem

If the capacity is not fully reserved, there is always a high enough demand for which increasing the proportion of shared capacity will be desirable for social costs.

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Conjecture (locally a demonstrated proposition)

While operating at social optimum, everything else being held constant, the optimal number of carpooling passengers increases with:

- Negative externalities of cars
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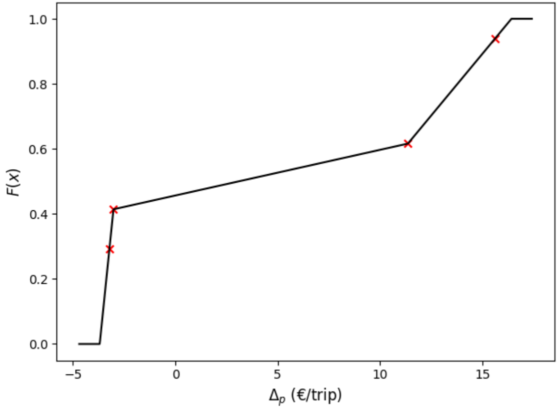
Proposition

While operating at social optimum, there is always a maximum profit $\Pi_{\max}$ the operator can make.
For a similar variations in car operation costs or externalities, the change in optimal modal split is the same, but the profit after the increase in externality is always lower than if the increase was in car operation.

Plan

A realistic example



Distribution of passenger inconvenience cost according to Le Goff et al. 2022

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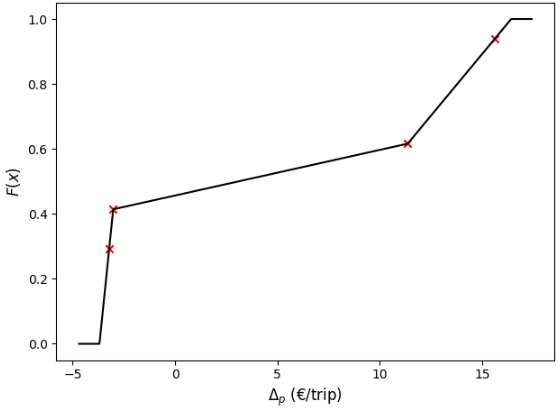

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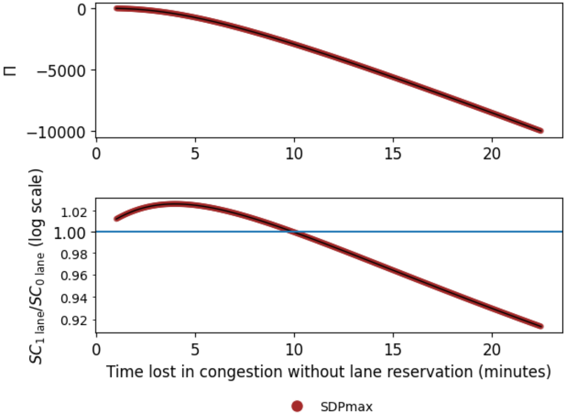

Outputs

Plan

Comparing to HOVL

Two potential effects:

- HOCL may reduce inconvenience costs compared to HOVL.

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Two potential effects:

- HOCL may reduce inconvenience costs compared to HOVL.
- HOCL and HOVL may have different structural effects. $\longleftarrow$**This as what we explore!**

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Similar existence properties. Same social cost function. But tighter constraints.

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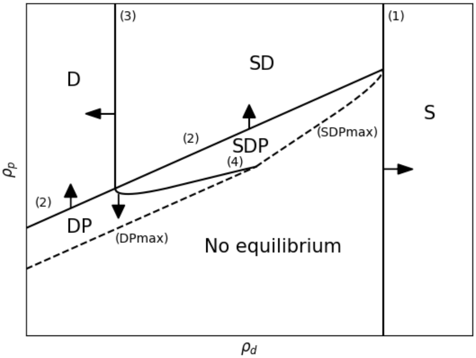

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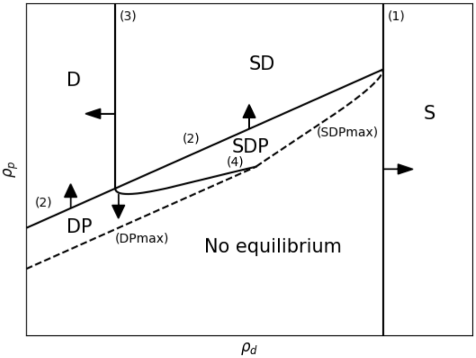

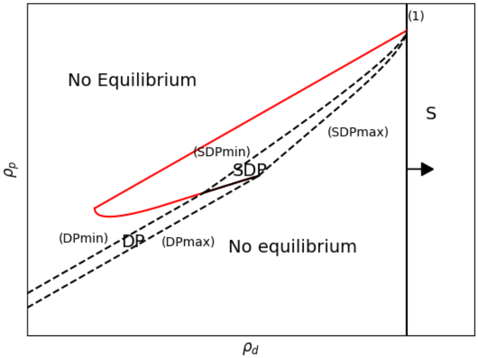

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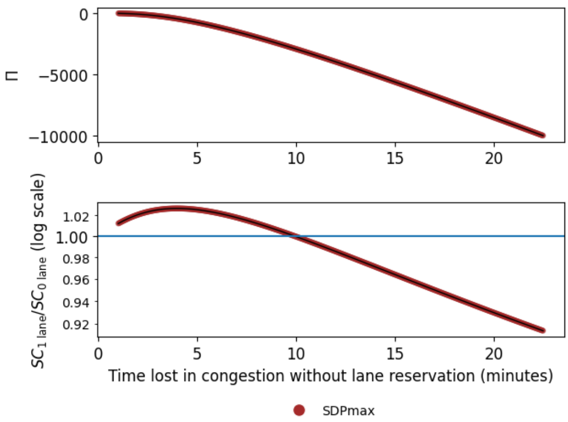
Remark

If the HOCL system is also able to reduce inconvenience costs compared to HOVL, Social Cost will be even lower.

Comparing to HOVL

Realistic example



Optimal of HOCL is reachable by HOVL.
Social costs are consequently the same.

Outputs (yes it is the same as before)

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2. Existence of equilibrium studied.
3. Intuitive but instructive mathematical properties demonstrated.
4. Numerical application with realistic values that show relevance of the system for realistic values of congestion.
5. The HOCL system performs better than classical HOVL, but not much difference in practice if inconvenience costs are kept equal.

THANK YOU!

Time for discussion

References I

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